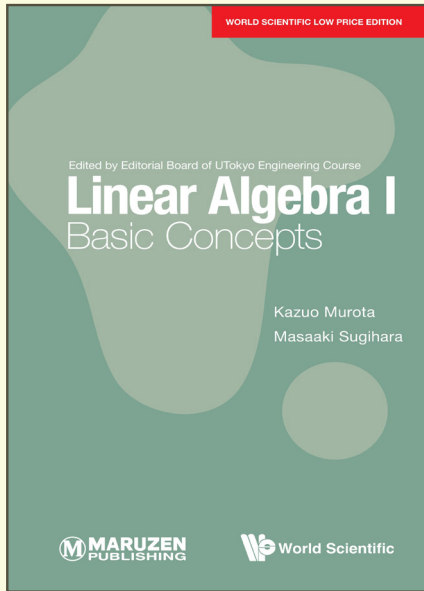# Linear Algebra I
Basic Concepts

By **Kazuo Murota**
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ABOUT THE BOOK

This is the first volume of the two-volume book on linear algebra, in the University of Tokyo (UTokyo) Engineering Course.

The objective of this volume is to present, from the engineering viewpoint, the standard mathematical results in linear algebra such as those on systems of equations and eigenvalue problems. In addition to giving mathematical theorems and formulas, it explains how the mathematical concepts such as rank, eigenvalues, and singular values are linked to engineering applications and numerical computations.

In particular, the following four aspects are emphasized.

- How matrices arise (discretization of differential equations, description of system structures, description of transition probability)
- What kinds of matrices arise (sparse matrices, positive definite matrices, diagonally-dominant matrices, nonnegative matrices, integer matrices, polynomial matrices)
- What characteristics we are interested in (rank, eigenvalues, singular values, positive definiteness)
- How we can compute (expansion formulas of determinants, elementary transformation, estimates of eigenvalues).

READERSHIP

Undergraduate students majoring in engineering and other mathematical sciences.

CONTENTS

- Preface
- Matrices
- Determinants
- Elementary Transformations and Elimination

- Rank
- Systems of Linear Equations
- Eigenvalues
- Quadratic Forms
- Singular Values and the Method of Least Squares
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ABOUT THE AUTHORS

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